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CRT-803

A Survey of Continuous Approximation Models in Logistics and Physical Distribution

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Centre de recherche sur les transports — Publication #803

May 1992

Dépôt légal – Bibliothèque nationale du Québec,
Bibliothèque nationale du Canada, 1992

93-01802/

ABSTRACT

We present a synthesis of the continuous approximation models that have been developed for logistics and physical distribution problems over the past thirty years. We propose a taxonomy for differentiating the problems and we classify them into nine different types of distribution problems using this taxonomy. Then a brief review of each paper is provided. This paper is aimed at providing practitioners and researchers with a tool to quickly and concisely inform them how a specific problem has been analysed by different people with a continuous approximation approach.

Keywords: logistics; physical distribution; continuous approximation.

RÉSUMÉ

Nous présentons dans cet article une synthèse des modèles d'approximation continue qui ont été développés durant les 30 dernières années pour les problèmes de logistique et de distribution physique. Nous proposons une taxonomie pour différencier les problèmes et nous classifions ceux-ci en neuf types différents de problèmes de distribution. Suit une description sommaire de chaque article. Cet article vise à procurer aux chercheurs un outil d'analyse.

Mots-clés: logistique; distribution physique; approximation continue.

1. INTRODUCTION

Logistics is broadly defined as the study of movement of goods and people in time and space. In commerce and industry, logistics is concerned with the providing of goods and services from a supply point to a demand point. The components of a logistics system are: supply, production and physical distribution (Figure 1). Physical distribution relates to the transportation and storage of goods from the plant to the consumers, possibly via some depots.

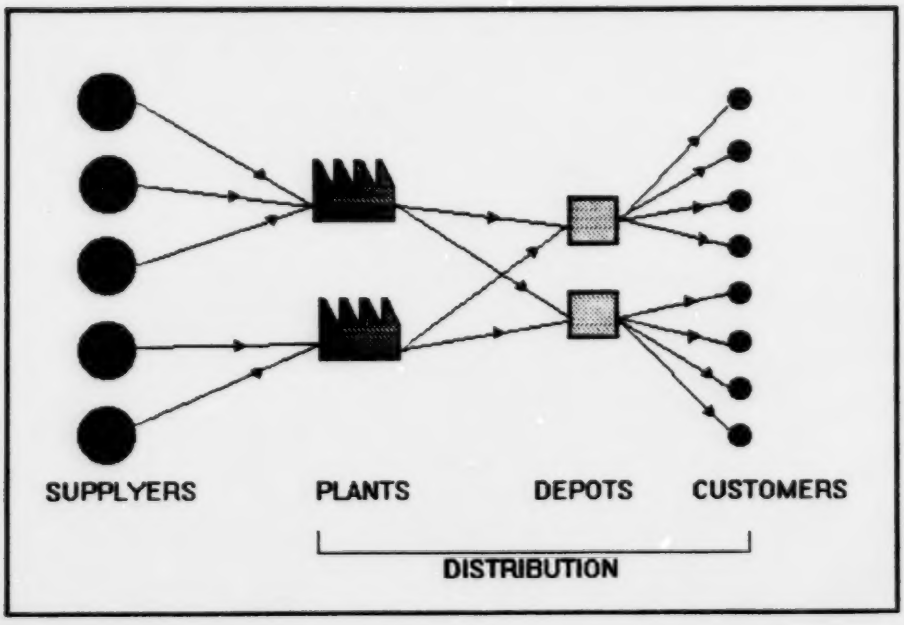

Figure 1: A logistics system

The issues to be addressed when planning or analysing a logistics system are related to the way of organizing the transportation and storage cost-efficiently. Costs that may be relevant include: handling and packaging costs; loading and transportation costs; space, machinery and maintenance costs; opportunity costs.

There are two main avenues that have been used to deal with logistics and distribution problems: the mathematical programming based approach and the continuous approximation approach. The former approach relies on detailed data and numerical methods whereas the latter relies on concise summaries and analytic models.

Continuous approximation models, which we address in this article, are based on the spatial density and the average distribution of the demand rather than on the precise location of every demand point. Daganzo (1991) notes that a main goal of this approach is to obtain reasonable solutions with as little information as possible and at the same time gain a clear understanding of the trade-offs at work. Continuous models can be useful when planning a new service or a possible expansion of an existing one. As they rely more on distribution functions than exact locations of customers, they do not require as much data as mathematical programming models. Their development and implementation are therefore easier and faster to achieve. Daskin (1985) reports that continuous models are being used 1) to model integrated logistics systems, 2) to devise robust models necessitating few data, and 3) to approximate discrete models.

The monograph of Eilon, Watson-Gandy and Christofides (1971) and that of Daganzo (1991) must be cited as two milestones in the development of continuous approximation models applied to logistics and distribution problems. Eilon et al.

address the problem of determining the number, size and location of depots and the problems of the design of delivery routes.

Daganzo (1991) is the first to attempt to present, in a more pedagogical manner, the continuous approximation method. He shows how to deal with the different logistics costs and takes the reader through the development of a number of continuous approximation models.

The aim of our article is a bit different from the work of Daganzo. Our goal is to provide practitioners and researchers with a tool to let them know quickly and in a concise manner how their problem has been analyzed by different people with a continuous approach. The objective is to show the relation between problems and techniques. With this article, we hope that the reader will get an overall view of the logistics problems that have been studied with continuous models over the last three decades.

2. HISTORICAL PERSPECTIVE

Continuous modeling is a technique used in operations research as a decision-making tool, particularly in strategic planning. This approach is not a very old idea, and so it is therefore surprising to find that continuous models had been applied much earlier to the movement of raw materials and products by Kantorovich (1942), Beckmann (1952), and later by Maejima (1979). Just prior to the 1960s, this technique was beginning to gain ground among research teams. Beardwood et al. (1959), for example, developed an analytical formula to estimate the total distance covered by a

traveling salesman in areas with a uniform density of destinations. This result was later verified and completed in important works carried out by Eilon et al. (1971), Stein (1978a) and Larson and Odoni (1981). The 1960s were dominated mainly by the work of one of the great pioneers of this technique, Reuben Smeed (1961, 1965, 1968), and by the work of John Wardrop (1969, 1971) in which continuous models were developed primarily to study pedestrian traffic in an urban area. Weiss (1985) later contributed his assessment of Smeed's work.

The use of continuous models for analyzing distances to be covered in connection with the location of service facilities, such as production suppliers, police stations and warehouses, has been treated by Beckmann (1968), and in the 1970s by Newell (1973, 1980). It was during the 1970s that new horizons opened up for a number of authors. Eilon et al. (1971), for example, used continuous models for analyzing the distribution problems of a depot serving an area with a uniform density of customers. The objective was to minimize mean traveling distance. The continuous approximation approach was used in other applications to estimate the response times of emergency vehicles (Larson and Stevenson, 1972; Kolesar and Blum, 1973).

Continuous models have also been considerably used to optimize public transportation systems with one-to-many or many-to-one travel demand pattern (i.e. from one origin to many destinations or from many origins to one destination) and with an inelastic demand (Vuchic and Newell, 1968; Newell, 1971; Byrne and Vuchic, 1972; Salzborn, 1972; Hurdle, 1973; Clarens and Hurdle, 1975; Byrne, 1976; Hurdle and Wirasinghe, 1980; Jansson, 1980; Wirasinghe, 1980; Tsao and Schonfeld,

1984; Wirasinghe and Seneviratne, 1986; Ghoneim and Wirasinghe, 1987; Kuah and Perl, 1988) or elastic demand (Kocur and Hendrickson, 1982; Oldifield and Bly, 1988; Chang and Schonfeld, 1989a,b). For many-to-many demand patterns, only a few studies on analytic optimization methods for transit systems have explicitly been considered especially for realistic spatial demand distributions with the inelastic demand (Holroyd, 1965; Vaughan and Cousins, 1977; Newell, 1979; Wirasinghe and Ghoneim, 1981; Vaughan, 1986) or the elastic demand (Chang and Schonfeld, 1989c). Generally, the objective function of most of those studies is the minimization of a total cost function consisting of the sum of operator costs and user costs. This paper is not concerned with those optimization analytic models for public transportation systems; however the Clarens and Hurdle (1975) model will be mentioned in our synthesis since it may be easily applied to problems of one-to-many physical distribution.

Those authors analyzed a problem of a public transportation system from a central terminal to destinations dispersed over a city and its suburbs. In this problem, the dispatcher must decide when and where to send each vehicle in order to minimize the operating costs of the vehicle and the costs of the users (i.e. in terms of the waiting time of passengers at the central terminal and the time spent on board the vehicle within a local service area). Daganzo et al. (1977) presented one of the first analytical approximation models for the many to one problem, one which could be applied to door to door transportation. Hendrickson (1978) did the same.

The first analytical model developed for many-to-many (i.e. from many origins to many destinations) distribution problems was presented by Daganzo (1978), the

results of which were improved in Daganzo (1984c). During the last decade, however, the use of continuous models has increased dramatically. A great deal of work has been carried out on the analysis of logistical systems, particularly physical distribution systems, in an effort to improve existing models on the one hand and, on the other, to widen the field of interest by developing new models. A number of recent studies have continued, therefore, to focus on one-to-many distribution problems with or without transhipments to minimize transportation and/or inventory costs, and the cost of service facilities using continuous models (e.g. Banks, Driscoll and Stanford, 1982; Daganzo, 1984a,b, 1987a,b; Han, 1984; Blumenfeld et al., 1985a,b,c, 1986; Burns et al., 1985; Daganzo and Newell, 1985, 1986; Cavalier and Sherali, 1986; Han and Daganzo, 1986; Campbell,1989a,b; Langevin, 1988; Langevin and Soumis, 1989; Erlenkotter,1989; Langevin and Saint-Mleux, 1991). It should be noted that the continuous models designed for these distribution problems are sometimes the same models as those used for the inverse (many-to-one) problem. However, this is not always the case. In some contexts, specific models are required, such as the Daganzo (1985, 1988a) and Hall (1985) models.

The literature also reveals that during the last decade several continuous models have been developed for many-to-many distribution problems (e.g. Hall, 1984, 1987a,b; Hall and Daganzo, 1984; Jordan and Burns, 1984; Daganzo, 1987c; Campbell, 1988, 1990).

3. CLASSIFICATION OF DISTRIBUTION PROBLEMS

In this section, some definitions are given which will be useful for the discussion that follows, as well as a taxonomy that will provide a method for differentiating between the various types of distribution problem.

3.1 Definitions

The *region* of the *request* for service is the geographical area served by one (or several) depot(s). This region may be divided into service *subregions* or *districts* containing a number of demand points. The service district may be further subdivided into service *zones* or *sectors*. These definitions are illustrated in Figure 2.

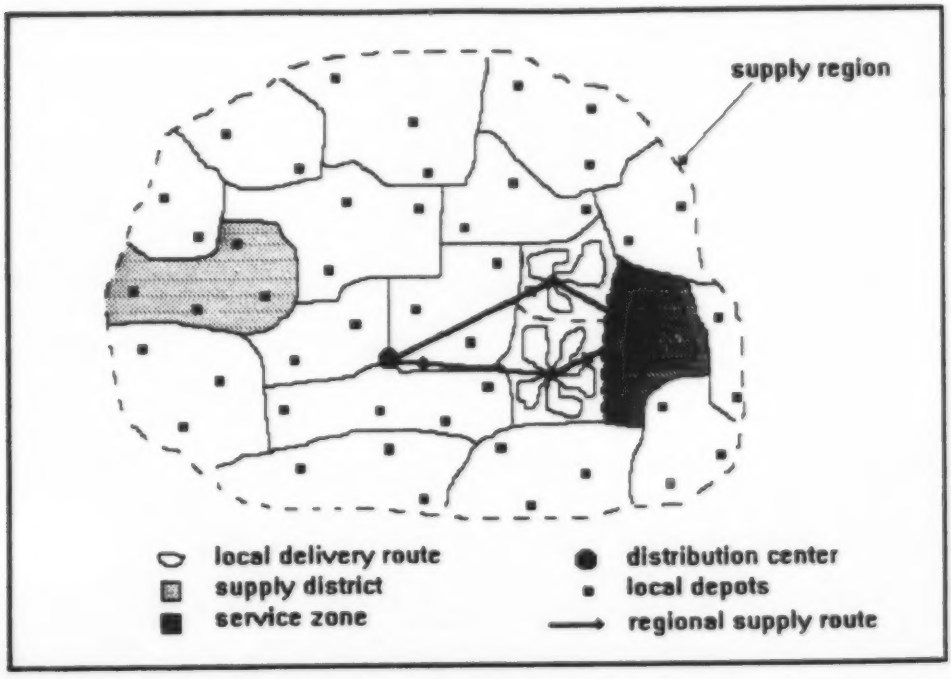

Figure 2: Distribution with one transshipment

A *strip strategy* is the routing strategy developed by Daganzo (1984a) by which a zone to be covered is cut into a strip of width w, where the demand points are visited along the strip in their longitudinal order (Figure 3). This strategy may be used when the vehicle leaves a depot outside this zone.

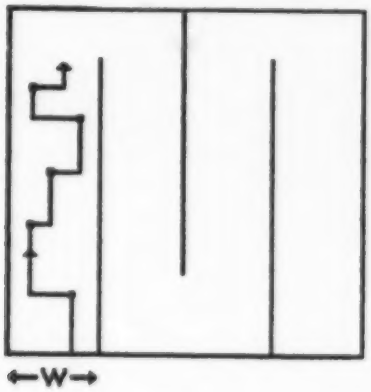

Figure 3: A strip strategy

When a single round trip is carried out within a zone, this is referred to as a *dual-strip* routing: a rectangular zone is divided into two strips of equal width and the service vehicle visits the demand points from one end of the strip to the other along both strips without backtracking (Figure 4).

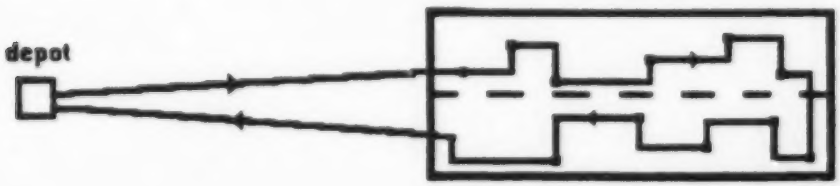

Figure 4: A dual-strip routing strategy

When the width of the zone corresponds to the width of the strip, the vehicle travels the route only once in the direction of the length of the strip, and this is called a *single-strip* routing (Figure 5): the demand points are visited only on the trip out, and the return to the depot begins from the other end of the strip once the farthest customer has been serviced.

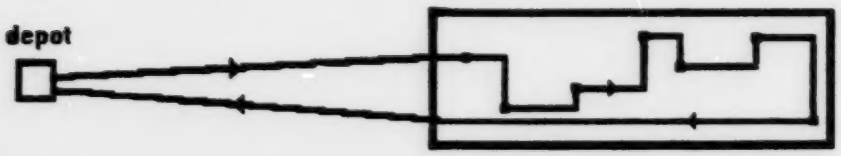

Figure 5: A single-strip routing strategy

Direct shipping is a strategy whereby product is shipped in separate lots directly to customers (one customer per trip), while *peddling* allows vehicles to visit more than one customer in a service district or zone before returning to the depot.

3.2 Taxonomy for differentiating distribution problems

Table 1 presents a taxonomy developed on the basis of three characteristics that are essential for describing a distribution problem. This taxonomy is designed to make it possible to distinguish between types of distribution problems. These characteristics are: an objective function, decision variables, and a system of parameters (see Brandeau and Chiu, 1989).

Using this taxonomy, we identify nine different types of distribution problem, with the appropriate references (Table 2).

Table 1: Taxonomy for differentiating distribution problems

I. Optimization objective
   1. Minimize mean traveled distance or
   2. Minimize mean traveling time or
   3. Minimize total distribution or logistics cost

II. Decision-making variables
   1. Location of warehouses
   2. Size or area of service zone
   3. Number of service vehicles
   4. Number of transshipment depots
   5. Volume (capacity) of service vehicle
   6. Average number of stops (customers) per service vehicle
   7. Frequency of (or time interval between) dispatching service vehicles/length of production cycle
   8. Other

III. System of parameters
   1. Topological structure:
      P = Plan; N = Network; O = Other
   2. Metric:
      E = Euclidean; L_1 = Manhattan; O = Other or not specified
   3. Number of service vehicles: N
   4. Number of depots: D
   5. Capacity of service vehicles
      L = limited, U = unlimited
   6. Routing strategy
      a) dual-strip routing
      b) single-strip routing
      c) direct shipping
      d) peddling
      e) cluster first, route second
      f) other

Table 2: Types of distribution problems

No.	Type of problem	References
P_1	One-to-many distribution	Beardwood et al., 1959; Eilon et al., 1971; Daganzo, 1984a,b; Blumenfeld and Beckman, 1985a; Newell, 1986; Newell and Daganzo, 1986a,b,; Hall, 1989.
P_2	One-to-many distribution with time constraints	Han, 1984; Han and Daganzo, 1986; Daganzo, 1987a,b; Langevin, 1988; Langevin and Soumis, 1989; Langevin and Saint-Mleux, 1991; Rosenfield et al., 1989; Klincewicz et al., 1990.
P_3	One-to-many distribution with transshipment and time constraints	Han, 1984; Han and Daganzo, 1989.
P_4	One-to-many distribution with transshipment	Campbell, 1989b; Erlenkotter, 1989.
P_5	One-to-many distribution with inventory (with or without transshipment)	Clarens and Hurdle, 1975; Banks, Driscoll and Stanford, 1982; Blumenfeld, Burns, Diltz and Daganzo, 1985b; Burns, Hall, Blumenfeld and Daganzo, 1985; Daganzo and Newell, 1985, 1986; Blumenfeld et al., 1986, 1987; Daganzo, 1988b; Campbell, 1989a.

Table 2: (Continued) Types of distribution problems

No.	Type of problem	References
P_6	Many-to-one distribution (or collection)	Daganzo, Hendrickson and Wilson, 1977; Hendrickson, 1978; Daganzo, 1988a; Mbaraga, 1990.
P_7	Many-to-one distribution (collection) with inventory	Daganzo, 1985; Hall, 1985.
P_8	Many-to-many distribution	Daganzo, 1978, 1984c; Stein, 1978a,b; Bélisle, 1989.
P_9	Many-to-many distribution with consolidation terminals	Hall, 1984, 1987a,b; Hall and Daganzo, 1984; Jordan and Burns, 1984; Daganzo, 1987c; Campbell, 1988, 1990.

4. SYNTHESIS OF PRINCIPAL DISTRIBUTION PROBLEMS

This section presents a synthesis of nine types of physical distribution problems that have been studied in the logistics domain using continuous approximation analytical methods. For convenience, these problems have been grouped in three categories: sub-section 4.1 reviews the one-to-many problems (P_1 to P_5), sub-section 4.2 the many-to-one problems (P_6 and P_7) and sub-section 4.2 the many-to-many problems (P_8 and P_9). Included is a formal description of each problem, and, for the corresponding references, the author's contribution is briefly summarized. In addition,

some relevant information characterizing each problem has also been summarized in Table 3, using the taxonomy presented in Table 1.

4.1 One-to-many physical distribution problems

P_1 One-to-many distribution

A number of continuous models have been developed for one-to-many physical distribution problems, with a routing component. The object here is to visit, from one depot, customers who are randomly and/or uniformly dispersed in a demand region in such a way as to minimize the total travelled distance or the total distribution cost. For these cases, an empirical formula is needed that links the length of an optimal route to the density of customers and the area of the demand region.

Beardwood et al. (1959) developed such a formula to approximate the length, L, of the optimal tour of a traveling salesman visiting N points distributed randomly and uniformly in a region of area A. The formula they developed was asymptotic, and the approximation of the length of tour, L, was $K\sqrt{AN}$, with N being fairly large and K a constant dependent on the metric used. Other analytical formulae may be found in Eilon et al. (1971) that express approximate distances in physical distribution problems (see Chapter 8).

Daganzo (1984a) later introduced his strip method (Figure 3), a systematic routing method enabling the determination of the quasi-optimal length of a traveling salesman's tour in irregularly shaped zones. The author shows that the optimal width of the strip, w, is equal to $\sqrt{3/\delta}$ for the Manhattan metric (L_1), and $\sqrt{2.95/\delta}$ for the

Table 3: Some characteristics of each problem type

No of problem type	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
Category system	one-to-many					many-to-one		many-to-many	
I. Objective	1,3	1,2,3	1,3	1,3	2,3	1,2	3	1,2	1,3
II. Decision variables	2,3,6	1,2,3,4,6	2,3	1,2	2,3,4,6,7	7,8	6,7	2,7,8	1,5,6
III. System of parameters									
(1) Topological structure	P, O	P	P	P	P, N	P	P	P	P, N
(2) Metric	E, L_1	E, L_1	E, L_1	E, L_1	E, L_1, O	L_1, O	E	E, L_1	E, L_1
(3) Number of vehicles					V			V	
(4) Number of depots				D	D				D
(5) Capacity of vehicles	L	U	U	L	L	L	L	L	L
(6) Routing strategy	a,b,d,e	a,b,d,e	a,d	c,d,f	c,d	d,f	d,e	a	c,d,f

Euclidean metric (L_2), where δ is the demand density (number of points per surface area unit). Further, the length of the optimal tour using this strategy is given by:

$$L = \begin{cases} 0,90\sqrt{AN} & \text{for the Euclidean metric} \\ 1,15\sqrt{AN} & \text{for the Manhattan metric} \end{cases}$$

Gaboune, Laporte and Soumis (1992) extend the work of Daganzo on the strip strategy to a variety of 2-D and 3-D metrics.

Elsewhere, Daganzo (1984b) obtains another formula for the approximate distance of a tour in the one-to-many physical distribution problem. Thus, with the strip strategy, each route contains a line-haul distance to reach the service zone, a local distance to be covered within the zone, and a back-haul distance to return to the point of origin (or the depot). These three steps are also part of a peddling route. By making:

$\tilde{\rho}$	=	the mean distance from the depot to the destinations,
δ	=	the density of stops (number of stops per surface area unit),
m	=	the average number of destinations visited per vehicle,
D	=	the length of a round trip,

Daganzo has shown that, when the number of stops is large, the expected value of the distance travelled is given by:

$$E(D) \approx 2\tilde{\rho} + m\left(0,57/\sqrt{\delta}\right).$$

Included here are the linehaul distance (round trip) and the total distance. This formula is useful in analyzing the trade-off between transportation and inventory

costs. Another formula for analyzing the trade-off between transportation costs and inventory costs has been developed by Blumenfeld and Beckmann (1985a). The authors analyzed the problem of physical distribution using the peddling approach, taking into account the spatial variations in the density and the demand of the destinations. They obtained analytical expressions for the length and number of stops in a tour to minimize the distribution costs based on the hypothesis that each tour services all of the clients in district.

Newell and Daganzo (1986a,b) and Newell (1986) propose approximation methods that facilitate the study of strategic planning for a fleet of delivery vehicles as a function of the distribution of the demand. These methods make it possible, in particular, to estimate the shape of the zone serviced by a vehicle of limited capacity. In Newell and Daganzo (1986a), the planned zones, in the case of a fleet of vehicles with limited capacity have an approximately rectangular shape which is elongated toward the source. In addition, all of the zones take the form of concentric rings, and within each service zone dual-strip routing is used (Figure 4). The radii of the rings and the number of zones in each ring are determined in order to minimize the total distance travelled by the vehicles, and an approximate length of the tour in each of the zones is obtained. It is evident from this article that the optimal width of a strip in an approximately rectangular zone of a concentric ring is equal to $(1{,}5\sqrt{\delta})$, where δ is the density (i.e. the number of points per surface area unit).

Hall (1989) examines the combined problem of loading and routing the vehicles with a view to minimizing the transportation costs in the one-to-many distribution

problem. The object in the first part of the problem is to load a minimum number of vehicles, while that of the second is to minimize the mean travelled distance per vehicle in one round trip. The customers are spread out in different delivery districts and several vehicles are loaded simultaneously for each district in order to favor efficient loading. The author presents a heuristic by which a number of vehicles are loaded simultaneously for certain destinations, which makes it possible to evaluate the number of vehicles and the length of the route per vehicle. The results show that in order to minimize the total cost of transportation, the best compromise between the efficient loading of vehicles and the length of the route per vehicle depends on the square root of the size of the region (or district) over which the loads are delivered and on the mean distance from the point of origin to the destinations.

P_2 One-to-many distribution with time constraints

When the delivery routes are subjected only to a time constraint, the design of the delivery zones of a demand region may be determined. This was so in the case studied by Han and Daganzo (1986) (see also Han, 1984), which was a (one-to-many) problem of distributing perishables without transshipments, where the capacity constraint of the vehicle was not considered. The authors analyzed and compared the two optimal strategies, single-strip and dual-strip, with a view to minimizing the total cost of distribution. Their results favored the single-strip strategy, except in the case where the service district is close to the depot.

Also in this category is the problem of vehicle routing with time windows (VRPTW) studied by Daganzo (1987a,b), Langevin (1988) and Langevin and Soumis

(1989) using the cluster first, route second strategy. In the articles by Daganzo (1987a,b), one work day of duration T is divided into p periods of time, and the time windows are modeled by specifying the time intervals during which a visit to one customer can be made. The clients are grouped into rectangular-shaped zones. Daganzo (1987a) has given the optimal dimensions of these zones, minimizing the travelled distance per customer visited, and has shown that this distance increases in proportion to $\sqrt{p}$ and that this increase is greater when the articles transported are small rather than large. In addition, the author compares the efficiencies of the various routing strategies within the delivery zones.

The case where the time windows are not specified for all customers was treated in Daganzo (1987b). He examined various distribution strategies and showed that a minimal traveling distance per customer is obtained when the two types of customer are treated differently.

The analytical method proposed by Langevin and Soumis (1989) enables division into delivery zones for the VRPTW, and involves partitioning the region into approximately rectangular zones that are arranged in concentric rings around the depot, each zone being assigned to a single vehicle, in order to minimize the transportation costs for a fleet of vehicles. Their method determines the optimal dimensions of a delivery zone as a function of its distance away from the depot and of the density of the demand in the region. It also enables, by an iterative process, determination of the zones from the periphery of a region towards the center where the depot is located. Routing for each of the vehicles may be achieved by using the algorithm designed

for the problem of the traveling salesman with time window constraints presented in Langevin (1988).

In Langevin and Saint-Mleux (1991), an interactive graphics method, based on analytical formulae, was introduced that enables the planner to progressively develop delivery zones on-screen, each of which is assigned to one vehicle. The analytical formulae used (Newell and Daganzo, 1986a; Langevin and Soumis, 1989) make it possible to determine the optimal dimensions of the delivery zones as a function of the speed of the vehicles, the density of the demand, how far away the zone is, and the maximum duration of a round trip or the maximum number of points that it is possible to visit.

In the article by Rosenfield et al. (1989), the problem of planning service districts with a time constraint is examined using an analytical approach based primarily on the initial and final traveling times and on the size of the service district. In this case, there are several service districts, each of which contains a service center (depot) at its center from which deliveries or visits are made at the premises of individual customers, uniformly dispersed. Thus, under certain hypotheses (constant speed, equal work days and random or predetermined demand, but with preassigned routes), the problem becomes one of determining the number and location of service centers. The authors have derived analytical expressions to determine the optimal number of service districts, which they then applied to the problem of planning delivery "units" for the United States postal system. They also presented a variation of the model for the case where the density of the service demand is variable.

Klincewicz et al. (1990) present a mathematical programming model for a single-source capacitated facility location problem. In a first step they use analytical methods for estimating parameters needed by the model. The model allows to choose between different delivery strategies and to determine the appropriate size of a private fleet assigned to a geographic area, with random demands, and served by a single warehouse. The geographic area is divided into sectors and the manager must choose the best way to serve each sector, be it by a private vehicle or by an outside carrier. The objective function incorporates the fixed and variable costs of a private vehicle, subject to a delivery duration constraint, and the outside carriers' delivery charges. The analytical methods are used because of the probabilistic nature of the demand. The variable number and location of the daily delivery points implies a variable route lenght and a variable amount of time required to serve these points by a private vehicle. The parameters (e.g. expected length of random routes) are estimated by analytical methods according to previous works (Blumenfeld and Beckman, 1985a; Burns et al., 1985; Christofides and Eilon, 1969; Daganzo, 1984a,b; Stein, 1978a; Vaughan, 1984).

P_3 One-to-many distribution with transshipment and time constraints

There is also the problem of physical distribution with transshipments and subject to a time constraint when two types of vehicles are used to minimize the total cost of distribution (Han, 1984; Han and Daganzo, 1989). These authors have analyzed this problem in the case of perishable products and have shown that the time constraint between production and delivery largely determines the design of the delivery system.

The service region is partitioned into districts, each of which contains zones. Each zone contains a single terminal located (approximately) at its center. With the dual-strip routing strategy, the approximation techniques used make it possible to cut each district into delivery zones of nearly optimal shape.

P_4 One-to-many distribution with transshipment

The problem of physical distribution with transshipments but without the time constraint, has also been studied, notably by Campbell (1989b) and Erlenkotter (1989). The object of the Campbell (1989b) model is to determine the optimal location and the optimal assignment of vehicles for a given number of terminals, while minimizing the mean transportation cost with two types of vehicles. The service region includes a number of sectors, each of which contains a terminal. The author analyzed direct shipping, where the line-haul vehicles transport separate loads directly from the point of origin to each terminal; local vehicles then ensure delivery of these separate loads directly from the terminals to each destination, i.e. the two types of vehicle make only one stop to deliver the product manufactured at the point of origin and which is the same for all of the demand destinations. Hypotheses designed to simplify the distribution process, such as locating terminals at the center of regions of the same size and processing a demand from the nearest terminal were also examined, but these do not constitute a general case. Analysis of the results has shown that the optimal design of the distribution system depends on the efficiency of the line-hauling, and that the locations of the terminals have no effect on the cost of transportation as long as the assignment of destinations is optimal.

Erlenkotter, who introduced the GOMA (General Optimal Market Area) model, has put together a fairly in-depth study of models, describing the fundamental trade-off between the economies of scale resulting from large service facilities (depots) and the high cost of transportation to markets farther away. The author also explored the use of his model for approximating large-sized discrete models, and provides, among other things, a list of pertinent references.

P_5 One-to-many distribution with inventory

Several articles on one-to-many distribution problems, including inventory costs, have been published. The objective in studying these problems is to minimize the total cost of distribution, in this case by analyzing the interdependencies and cost trade-offs related to the various logistical components. These include, in particular, product transportation and storage. Authors like Clarens and Hurdle (1975) and Burns, Hall, Blumenfeld and Daganzo (1985) have studied this type of problem in order to determine the trade-off between transportation and inventory costs, with an optimal number of districts and vehicles per district. The Clarens and Hurdle (1975) model, however, is based on the assumption that the number of stops per surface area unit is a known quantity, while the Burns et al. (1985) study is based on the assumption that the number of stops per vehicle round trip is known. The latter have analyzed and compared the two delivery strategies, direct shipping and peddling, with a view to minimizing the total cost of distribution, and their results show that peddling is a more appropriate means of distribution than direct shipping when the articles are valuable.

The model of Blumenfeld et al. (1985b) applies on distribution networks with

a consolidation terminal and in a "few-to-many" system, i.e. from few origins to many destinations. This is also the case with Blumenfeld et al. (1987) in which a method is develop to help GM's Delco Electronics Division to determine the best shipment sizes and routes for its products. The aim of this paper is to develop concrete and comprehensible means to see how transportation and inventory costs depend on shipment sizes and routes, to understand the trade-offs between transportation and inventory costs and to analyze the sensitivity of costs to systems conditions.

It should be noted that the Clarens and Hurdle model, developed specially for a mass transit bus system serving a city and its suburbs from a central terminal to dispersed destinations, may be easily applied to problems of physical distribution from a depot to a fixed number of points of sale. The authors have considered time dependent demand in analyzing a commuter bus system with inelastic demand; however, analytic solutions were obtained only when the system parameters were assumed to be independent of time.

In a similar vein, Banks, Driscoll and Stanford (1982) developed a model that is essentially an extension of the Clarens and Hurdle (1975) approach. This model also makes it possible to simultaneously evaluate the operating costs and the traveling time associated with a passenger transit system. They then studied the case in which there are optimal numbers of vehicles and districts, on the assumption that each passenger transported makes one stop. As well they assumed that within each district distribution is not performed according to an efficient routing strategy, which, in turn, affects the routing configuration used. Daganzo and Newell (1985) also analyzed this

case, but they adopted a different routing strategy, assuming inventory to be constant in one scenario and variable in another; they assumed, in addition, that each article transported necessitates one stop. These authors showed to what extent the nature of the articles transported influences the optimal configuration of the distribution system and the total cost of distribution. Their results constitute a direct application of the Clarens and Hurdle (1975) model.

In another article, Daganzo and Newell (1986) proposed a flexible continuous approximation method for planning distribution systems that include many transshipments, with a view to minimizing transportation costs plus the inventory carrying cost. The proposed method enables a service region to be divided into sectors served by a single depot. Whereas the number of transfers increases with the distance from the point of origin, destinations that are very close to this point are served without transshipments. The result is a hierarchy of terminals with more levels for the greater distances. This method embodies several essential characteristics for a nearly optimal solution to this problem, rather than providing a single solution.

Daganzo (1988b) examines merchandise distribution systems (for the same one-to-many problems), based on the assumption that the merchandise is homogeneous and cheap, and that there is no seasonal effect or any restriction on the length of the route. The author proceeds by comparing two approaches to merchandise consolidation that would minimize transportation and inventory costs: the consolidation of merchandise in vehicles, without transfers, and with peddling; the use of another strategy that allows for transfers to local depots. His analysis showed that under certain specific

conditions transshipments are not necessary, except in the case of many-to-many distribution systems (see Daganzo, 1987c).

The trade-offs between transportation, inventory and production costs and merchandise distribution networks are amply illustrated in an analysis by Blumenfeld, Burns, Diltz and Daganzo (1985b). In this article, they propose the determination of optimal strategies involving routes and sizes of vehicle loads between points of origin and destinations by analyzing these trade-offs. The strategies the authors considered for distribution were direct shipping and shipping via a consolidation terminal, and, finally, a combination of these two strategies. The standard Economic Order Quantity (EOQ) methods (Arrow et al., 1958; Magee and Boodmann, 1967) are used to examine the interdependence of costs for the networks that include direct shipping. For all three strategies, coordinated independent production and merchandise transportation planning are both analyzed in detail. The resulting model does not take into account fixed operating costs, the depot construction cost, or the capacity constraints of the depot; it is, however, a useful tool in logistics planning.

While Erlenkotter (1989) had explored the trade-off between transportation costs and the cost of facilities using continuous models, Campbell (1989a) developed another analytical model. This model incorporates all three costs (transportation, inventory and terminal costs) for the one-to-many distribution problem with two types of delivery vehicles. Campbell proposes a service region partitioned into districts, each containing a terminal, where the line-haul vehicles ensure bulk delivery direct from the point of origin to the terminals, while local vehicles carry out local deliveries,

following peddling routes and with stops at various different destinations. He analyzes two types of trade-off at once: the trade-off between transportation and inventory costs and the trade-off between transportation costs and terminal-related costs. This analysis provides optimal values for the number of terminals, the number of stops for local vehicles and the size of the loads of line-haul vehicles required to minimize the total distribution cost. The author also explores the possibility of reducing the cost of distribution by a clever manipulation of the system involving the coordination of the loads at the entrances and exits of the terminals. This can reduce waiting time and, consequently, decrease the total cost of distribution.

4.2 Many-to-one distribution problems

These are actually supply (or collection) problems from a number of suppliers at the same destination (factory, depot, etc.). Although most of the time the models obtained for one-to-many distribution problems are easily applicable to many-to-one problems (e.g. Blumenfeld, Burns, Diltz and Daganzo (1985b); Burns, Hall, Blumenfeld and Daganzo (1985); Campbell (1989b)), a few continuous models have been developed specifically for this particular problem of collecting from several suppliers.

P_6 Many-to-one distribution

The Daganzo, Hendrickson and Wilson (1977) model is one of the first of these, and provides, as pointed out by Bélisle (1989), an estimate of the mean route length and waiting times of the user in a system where the service vehicle visits a fixed location periodically, which corresponds to a common destination (or common point

of origin, depending on the circumstances) of all the movements carried out in the service zone. This model makes it possible to determine how often the vehicle must visit the fixed location in order to minimize the mean total time spent by a user in the system. The model was validated using simulated and real data. Closed form solutions for waiting time were derived with the assumption that each equalized subarea is served by only one vehicle.

Daganzo (1988a) analyzes the problem of shipping items of differents forms, shapes, or sizes in order to reduce as much as possible the vehicle-miles travelled. He first considers the building block in which items are sent directly from only one origin to one destination, without using consolidation terminals. In this case, the objective function (vehicle-miles of travel) is minimized by dispatching as few vehicles as possible. Secondly, he examines the situation with many origins and one destination, and one consolidation terminal. He then analyzes the role of the consolidation terminal and shows that it can provide a reduction in the total number of vehicle-miles travelled per unit time when using adequately and in optimal proportions load mixtures. The vehicle-miles include in this case : 1) direct miles, linking directly each origin with the destination, 2) inbound miles to the consolidation terminal, linking the origins with the consolidation terminal, and 3) outbound miles from the consolidation terminal, linking the consolidation terminal with the destination.

The problem of supplying several sources at a single destination with capacity and time constraints is discussed in Mbaraga (1990). The author, basing his argument on previous work (Langevin and Soumis, 1989; Han and Daganzo, 1989), examines the

problem of collecting perishables from several suppliers in one region and delivering them to a single destination (factory) within a fixed period of time and with trucks of limited capacity. Each truck is assigned to a zone of the region where the collection is to be made and then returns to the factory, without exceeding the preset maximum time allowed. The analytical model developed provides an almost optimal division of the collection region into zones to minimize the total cost per unit load transported, including the operating costs of the trucks and the fixed costs of stops at the suppliers' premises, which are uniformly dispersed over the collection region.

P_7 Many-to-one distribution with inventory

Now, the problem of collection or supplying may include an inventory of the loads on-board vehicles, and there may be a time constraint associated with the shipping time of the vehicles carrying out the collection (P_7). It is for cases such as these that the Daganzo (1985) and Hall (1985) models were developed. The object of the Daganzo model is to minimize costs related to the transportation of heterogeneous articles to be delivered to a common destination. This model enables determination of the optimal number of transport systems, and the assignment of individual suppliers to each system. The Hall model, which is based on the results of the Daganzo model, is designed to determine the optimal frequency of dispatching vehicles when a large number of heterogeneous suppliers is visited more often than a small number, in order to minimize the costs associated with the collection.

4.3 Many-to-many distribution problems

P_8 Many-to-many distribution

A review of the literature reveals that few continuous models have been developed for many-to-many distribution problems. The first models of this type are to be found in Daganzo (1978) and Stein (1978a,b). The article by Daganzo (1978) analyzes three routing strategies, under certain hypotheses, and evaluates for each the mean route and waiting times of a user when the service zone is served by M vehicles. One of the hypotheses that the author puts forward is that the vehicles are never in competition with each other for the right to service the same demand, while at the same time they are always capable of serving the demand that is closest to their point of origin. The results obtained with the model under this hypothesis underestimate the real times, however. The hypothesis has, in fact, a fundamental shortcoming, compounded in Daganzo (1984c), which is its use of a very simple routing strategy. This was prompted by earlier results (Daganzo, 1984a), which in this case enables the establishment of a rule of cooperation among the vehicle operators preventing any kind of competition.

The articles by Stein (1978a,b) are based on the previous results, in which the length of one tour visiting N points distributed uniformly and randomly in a region of surface area A is approximated to $K\sqrt{AN}$, where K is a constant (see Beardwood et al., 1959; Eilon et al., 1971). The author analyzes the length of the optimal round trip of a vehicle that must service N demands, each of which has one point of origin and one destination. He uses Monte-Carlo simulations to estimate the constant K to

0.765, and also presents lower and higher bounds for the length of an optimal tour of N demand points. This optimal length has a lower bound of $K\sqrt{2AN}$ (optimal length of a tour over 2N points) and an upper bound of $2K\sqrt{AN}$ (length of an initial tour over the N points of origin, followed by a second tour over the N destinations).

The work published by Bélisle (1989), which has a similar focus to that of Daganzo (1978, 1984c), proposes analytical approximation models enabling estimation of the mean service time of a user of a door-to-door transport system based on two routing strategies (unidirectional and bidirectional sweeping) in a service zone assumed to be rectangular (see Daganzo, 1984c), in which the points of origin and the destinations of the movements are distributed uniformly. In addition, the destination of the movement is independent of the point of origin for a given demand.

P_9 Many-to-many distribution with consolidation terminal

The use of terminals for consolidation of shipments in many-to-many logistics networks allows to minimize the mean travelled distance or the total costs including inventory costs. One can find in the literature a few models (Hall, 1984, 1987a; Daganzo, 1987c) for cases where the goods can be shipped through the closest terminal of either the origin or the destination, or through both of them.

Hall (1984) presents a model to analyze the trade-off between the mean travelled distance and the number of terminals based on the preceding two strategies. He uses a rectangular grid metric. This model shows that a growth in the number of terminals makes the mean travelled distances decrease slowly.

Hall and Daganzo (1984), Daganzo (1987c), and Hall (1987a) have looked at a number of routing strategies when the terminals are located at the center of identical regions, with a L_1 metric. Hall (1987b) analyzes a freight transportation problem on a network with concave costs. He develops an optimization procedure allowing to find optimal routes for freight on a network with many origins, few destinations and one terminal where each origin ships a different "commodity" to each destination and when arc flow capacities are not considered.

Jordan and Burns (1984) develop two backhauling models (continuous and discrete models) by considering two terminals each sending full truckloads to many customers under steady-state conditions. Their study concerns small networks and aims at minimizing empty truck-miles using strategies where each vehicle visits at most two terminals before returning home. The continuous backhaul model developed, using simplifying assumptions about customers that must be continuously distributed over a region, and travel distances, examines the impact of terminal locations and imbalances in the number of loads sent by terminals on backhauling opportunities.

Finally, Campbell (1988), based on these previous works, analyzes three strategies of routing using consolidation terminals. In this model, merchandises are first shipped by a local vehicle from their origin to a terminal, then by a line-haul vehicle between terminals, and at last by a local vehicle from the terminal to the destination. At least one terminal is visited to ensure consolidation. The consolidation of shipments favors highly efficient shippings between terminals.

Campbell (1990) develops a continuous approximation model of a general freight carrier distribution in a fixed region with an increasing density of demand. The model analyzes the tradeoffs between transportation, terminal and relocation costs. The author explores a few dynamic strategies for updating terminal locations under some scenarios and, from the optimal strategy, the upper and lower bounds on the distribution cost are provided.

As a final part of this section, we mention two papers that integrate continuous approximation models and mathematical programming. Robusté et al. (1990) use a strip strategy to find an initial solution to TSPs and VRPs and then improve the solution by simulated annealing. Hall et al. (1991) propose an integrated routing algorithm in three steps: 1. An initial assignment of the customers to the vehicles is done by dividing the region into rings and then the ring into sectors with a modified SWEEP algorithm. 2. A reassignment is done by a generalized assignment algorithm. 3. A TSP is solved on each vehicle.

5. Conclusion

There are two main approaches for solving logistics and distribution problems: the mathematical programming approach, which relies on detailed data, and continuous approximation models, which rely on summaries of data. In this paper we have surveyed around a hundred continuous approximation papers published in the last three decades. This survey enables us to arrive at a very positive judgment on the state of the field. Although the mathematical programming and the continuous approximation approaches have had very few interactions in the past, we feel that

the trend in the coming years will be to develop combined methods. The continuous approximation methods could be used as "start-up" for more detailed optimization techniques. Already, Robusté et al. (1990) and Hall et al. (1991) have begun to explore this avenue.

We hope that this paper will be a useful tool for researchers. It has been designed to provide a quick reference guide for anyone working on a specific logistics and distribution problem. For the reader looking for more details on the *technique* of continuous approximation modeling, we strongly recommend the monograph of Daganzo (1991).

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